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The Møller Energy Complexes of Various Wormholes in General Relativity and Teleparallel Gravity

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This study is aimed to elaborate the energy problem of general wormhole space-times in two different approaches of gravity such as general relativity and teleparallel gravity. In this connection, the energy for well-known wormhole space-times is evaluated using Møller energy-momentum prescription in these different approximations. We obtained that energy distributions of Møller definition give the same results for various wormhole space-times in general relativity (GR) and teleparallel gravity (TG). The results strengthen the importance of Møller energy-momentum definitions in given space-times and viewpoint of Lessner that Møller energy-momentum complex is a powerful concept for energy and momentum.

KEY WORDS: Wormholes; Energy-momentum distributions; Møller prescription; General theory of gravity and Teleparallel gravity.

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1. INTRODUCTION

It is well known that one of the most interesting and challenging problems of general relativity is the energy and momentum localization. Energy-momentum is an important conserved quantity in any physical theory whose definition has been under investigation for a long time from the General Relativity viewpoint. The problem is to find an expression which is physically meaningful. The point is that the gravitational field can be made locally vanish and so one is always able to find the frame in which the energy-momentum of gravitational field is zero while in the other frames, it is not true. Unfortunately, there is still no generally accepted definition of energy-momentum for gravitational field. The problem arises with the expression defining the gravitational field energy part.

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In the theory of General Relativity, the energy-momentum conservation laws are given by

$$T^b_{a;b} = 0, \quad (a, b = 0, 1, 2, 3), \tag{1}$$

where T^b_a denotes the energy-momentum tensor. In order to change the covariant divergence into an ordinary divergence so that global energy-momentum conservation, including the contribution from gravity, can be expressed in the usual manner as in electromagnetism, Einstein formulated (Møller, 1957) the conservation law in the following form

$$\frac{\partial}{\partial x^b}\left(\sqrt{-g}\left(T^b_a + t^b_a\right)\right) = 0. \tag{2}$$

Here t^b_a is not a tensor quantity and is called the gravitational field pseudo-tensor. Schrödinger showed that the pseudo-tensor can be made vanish outside the Schwarzschild radius using a suitable choice of coordinates. There have been many attempts in order to find a more suitable quantity for describing the distribution of energy and momentum due to matter, non-gravitational and gravitational fields. The proposed quantities which actually fulfill the conservation law of matter plus gravitational parts are called gravitational field pseudo-tensors. The choice of the gravitational field pseudo-tensor is not unique. Because of this, quite a few definitions of these pseudo-tensors have been proposed. The notion of energy-momentum prescriptions was severely criticized for a number reasons. Firstly, the nature of symmetric and locally conserved object is non-tensorial one; thus its physical interpretation appeared obscure (Chandrasekhar and Ferrari, 1991). Secondly, different energy-momentum complexes could yield different energy-momentum distributions for the same gravitational background (Bergqvist, 1992). Finally, energy-momentum complexes were local objects while it was generally believed that the suitable energy-momentum of the gravitational field was only total, i.e. it cannot be localized (Chen and Nester, 1999). There have been several attempts to calculate energy-momentum prescriptions associated with different space times (Virbhadra and Rosen, 1993; Virbhadra and Chamorro, 1995).

In order to obtain a meaningful expression for energy, momentum and angular momentum for a general relativistic system, Einstein himself proposed an expression. After Einstein's energy-momentum complex (Trautman, 1962), many complexes have been found, for instance, Landau and Lifshitz (1987), Tolman (1934), Papapetrou (1948), Møller (1958, 1961), Weinberg (1972) and Bergmann and Thomson (1953). Some of these definitions are coordinate dependent while others are not. There lies a dispute on the importance of non-tensorial energy-momentum complexes whose physical interpretations have been questioned by a number of physicists, including Weyl, Pauli and Eddington. Also, there exists an opinion that the energy-momentum pseudo-tensors are not useful to find meaningful results in a given geometry. Ever since the Einstein's energy-momentum complex (Einstein, 1915), used for calculating energy and momentum in a general relativistic system, many attempts have been made to evaluate the energy distribution for a given space-time (Tolman, 1934). Except for the one which was defined by Møller, these definitions only give meaningful results if the calculations are performed in "Cartesian" coordinates. Møller constructed an expression which enables one to evaluate energy and momentum in any coordinate system. Lessner (1996) argued that the Møller prescription is a powerful concept of energy-momentum in general relativity.

In this paper, we calulate the Møller energy distribution of the well-known wormhole space-times (zero density wormhole metric, zero radial tides wormhole metric., conformal wormhole metric, inflating wormhole metric, wormhole metrics in Friedmann–Robertson–Walker (FRW) cosmology and Visser–Kar–Dadich wormhole metric) using general non-static spherically symmetric metric in general relativity and teleparallel gravity. Recently, Virbhadra (1999) investigated whether or not the energy-momentum complexes of Einstein, Landau and Lifshitz, Papapetrou and Weinberg give the same energy distribution for the most general non-static spherically symmetric space-time. Furthermore, Xulu (2003) computed the Møller energy distribution of given space-time in general relativity and compared this results with one obtained by Virbhadra in the Einstein prescription. The general spherically symmetric metric space-time is described by the line element.

$$ds^2 = A^2(r,t)\,dt^2 - B^2(r,t)\,dr^2 - C^2(r,t)r^2(d\theta^2 + \sin^2\theta\,d\phi^2) \tag{3}$$

This includes the following well-known wormhole space-times as special cases: The static wormhole metric, zero density wormhole metric, zero radial tides wormhole metric, conformal wormhole metric, inflating wormhole metric, wormhole metric with electric charge, wormhole metric with scalar field, wormhole metrics in Friedmann–Robertson–Walker (FRW) cosmology and Visser–Kar–Dadich wormhole metric. Throughout this paper, we use units where $G = \hbar = c = 1$. Greek and Latin indices run from 0 to 3 and represent the vector components and the vector numbers, respectively.

We will proceed according to the following scheme. In Section 2, we give simple definition of Møller energy prescription of wormholes in general relativity and teleparallel gravity. In Section 3, we get Møller energy distributions of various wormholes in general relativity and teleparallel gravity. Finally, we summarize and discuss our results.

2. MØLLER ENERGY-MOMENTUM COMPLEXES OF GENERAL SPHERICAL SYMMETRIC SPACE-TIMES IN GENERAL RELATIVITY (GR) AND TELEPARALLEL GRAVITY (TG)

2.1. Møller Energy-Momentum Prescription of General Spherical Symmetric Space-times in GR

In general relativity, Møller energy-momentum complex (Møller, 1978) is given by

$$\Theta_\mu^\nu = \frac{1}{8\pi}\Omega_{\mu,\sigma}^{\nu\sigma} \tag{4}$$

where the antisymmetric superpotential is

$$\Omega_\mu^{\nu\sigma} = -\Omega_\mu^{\sigma\nu} = \sqrt{-g}\left(\frac{\partial g_{\mu\alpha}}{\partial x^\beta} - \frac{\partial g_{\mu\beta}}{\partial x^\alpha}\right) g^{\nu\beta} g^{\alpha\sigma} \tag{5}$$

Θ_0^0 is the energy density and Θ_μ^0 are the momentum density components. Also, the energy-momentum complex Θ_μ^ν satisfies the local conservation laws:

$$\frac{\partial\Theta_\mu^\nu}{\partial x^\nu} = 0 \tag{6}$$

Obviously, the energy and momentum of the physical system in four-dimensional background is given by

$$P_\alpha = \iiint \Theta_\alpha^0 dx^1\, dx^2\, dx^3 \tag{7}$$

where P_0 and P_α denote for the energy and the momentum components, respectively. The energy component is obtained by using the Gauss theorem

$$E^{GR} = \frac{1}{8\pi}\iint \Omega_0^{0\sigma}\eta_\sigma\, dS \tag{8}$$

where η_σ is the outward unit normal vector over an infinitesimal surface element dS.

The nonvanishing required component of Møller's superpotential for the line element given by Eq. (3) is

$$\Omega_0^{01} = \frac{2A_{,r}\, C^2 r^2\, \sin\theta}{B} \tag{9}$$

Using the above component in Eq. (8), we obtain the energy in the form

$$E^{GR} = \frac{A_{,r}\, C^2 r^2}{B} \tag{10}$$

These results are consistent with Xulu's (2003) results.

2.2. Møller Energy-Momentum Prescription of General Spherical Symmetric Space-times in TG

Møller (1978) modified general relativity by constructing a new field theory in the tetrad space. He was able to find a general expression for an energy-momentum complex (Møller, 1958) Ξ_{μ}^{ν} that possesses all the required satisfactory properties and formed its super-potential $\Upsilon_{\mu}^{\nu\alpha}$ using the method of infinitesimal transformations:

$$\Xi_{\mu}^{\nu} = \Upsilon_{\mu,\sigma}^{\nu\sigma} \tag{11}$$

where the expression for the superpotential of Møller's theory can be written in the form

$$\Upsilon_{\mu}^{\nu\sigma} = \frac{(-g)^{1/2}}{2\kappa} P_{\alpha\beta\rho}^{\lambda\nu\sigma} (\Phi^{\beta} g^{\rho\alpha} g_{\mu\lambda} - \Lambda g_{\lambda\mu} \gamma^{\alpha\beta\rho} - (1-2\Lambda) g_{\lambda\mu} \gamma^{\rho\beta\alpha}) \tag{12}$$

where Λ equals to a free dimensionless parameter of teleparallel gravity and κ is the Einstein constant. Also, $P_{\alpha\beta\rho}^{\lambda\nu\sigma}$ is

$$P_{\alpha\beta\rho}^{\lambda\nu\sigma} = \delta_{\alpha}^{\lambda} \xi_{\rho\alpha}^{\nu\sigma} + \delta_{\beta}^{\lambda} \xi_{\rho\alpha}^{\nu\sigma} - \delta_{\rho}^{\nu} \xi_{\alpha\beta}^{\nu\sigma}, \tag{13}$$

$\xi_{\beta\rho}^{\nu\sigma}$ is the tensor

$$\xi_{\beta\rho}^{\nu\sigma} = \delta_{\beta}^{\nu} \delta_{\rho}^{\sigma} - \delta_{\rho}^{\nu} \delta_{\beta}^{\sigma}, \tag{14}$$

Φ_{α} is the basic vector defined by

$$\Phi_{\alpha} = \gamma_{\alpha\beta}^{\beta} \tag{15}$$

and the central role in Møller's theory is played by tensor

$$\gamma_{\mu\nu\sigma} = e_{m\mu} e_{m\nu;\sigma} \tag{16}$$

here semicolon denotes covariant differentiation using the Christoffel symbols. $e_{m\nu}$ is the tetrad field and defined uniquely by $g^{uv} = e_i^u e_j^v \eta^{ij}$ where η_{ij} is the Minkowski metric. Finally, the energy in teleparallel gravity is expressed by the surface integral

$$E^{TG} = \lim_{r\to\infty} \int \Upsilon_0^{0\alpha} n_{\alpha}\, dS \tag{17}$$

with n_{α} being the unit three-vector normal to surface element dS.

The general form of the tetrad, e_i^{μ} having spherical symmetry was given by Robertson (Robertson, 1932). In the Cartesian form it can be written as

$$e_0^0 = iA, \qquad e_a^0 = Cx^a, \qquad e_0^{\alpha} = iDx^{\alpha},$$
$$e_a^{\alpha} = B\delta_a^{\alpha} + Ex^a x^{\alpha} + \varepsilon_{a\alpha\beta} F x^{\beta} \tag{18}$$

where A, B, C, D, E and F are the functions of t and r. The zeroth vector e_0^α has the factor $i^2 = -1$ to preserve Lorentz signature. The tetrad of Minkowski space-time is $e_a^\alpha = \text{diag}(i, 1, 1, 1)$.

Using the general coordinate transformation

$$e_{a\beta} = \frac{\partial X^{\alpha'}}{\partial X^\beta} e_{a\alpha} \tag{19}$$

where X^β and $X^{\alpha'}$ are the isotropic and Schwarzschild coordinates (t, r, θ, ϕ) (Salti, 2006). If we would like to calculate Møller Energy Complexes all the general metric, given by Eq. (3), it is needed to calculate tetrad components of line element. So, we obtain the tetrad components of line element in Eq. (3) as

$$\begin{bmatrix} i/A & 0 & 0 & 0 \\ 0 & \sin\theta\cos\phi/B & \cos\theta\cos\phi/Cr & -\sin\phi/Cr\sin\theta \\ 0 & \sin\theta\sin\phi/B & \cos\theta\sin\phi/Cr & \cos\phi/Cr\sin\theta \\ 0 & \cos\theta/B & -\sin\theta/Cr & 0 \end{bmatrix} \tag{20}$$

Hence, the required non-vanishing component of $\Upsilon_\mu^{\nu\sigma}$ for the line element given by Eq. (3) is

$$\Upsilon_0^{01} = \frac{2A_{,r}C^2r^2\sin\theta}{B} \tag{21}$$

Using the Eqs. (21) and (17), the energy of Møller is obtained in teleparallel gravity as

$$E^{TG} = \frac{A_{,r}C^2R^2}{B}. \tag{22}$$

3. MØLLER ENERGY COMPLEXES OF VARIOUS WORMHOLES IN GR AND TG

The wormhole solutions of the Einstein equations started with Einstein himself, since he was interested in giving a field representation of particles (Einstein and Rosen, 1935). The idea was further developed by Ellis (1973) and others, where instead of particles, they try to model them as "bridges" between two regions of the space-time. The idea of considering such solutions a actual connections between two separated regions of the Universe has attracted a lot of attention since the seminal work of Morris and Thorne (1998). For the Lorentzian wormhole to be traversable, it requires exotic matter which violates the known energy conditions. To find the reasonable models, there had been studying on the generalized models of the wormhole with other matters and/or in various geometries. Among the models, the matter or wave in the wormhole geometry as the primary and auxiliary effects (Kim, 2000). Recently, the solution for the electrically charged case was also found (Kim, 2001).

In this section, we consider Eqs. (9) and (21) with Eqs. (10) and (22) to find exact solutions for the energy distributions associated with the various wormhole models in general relativity and teleparallel gravity.

3.1. Møller Energy of Zero Radial Tides Wormhole in GR and TG

If we replace $A^2(r,t) = 1$, $B^2(r,t) = \frac{1}{1-b(r)/r}$ and $C^2(r,t) = 1$ in the line element given by Eq. (3), nonstatic spherically symmetric metric transforms to the "zero radial tides" wormhole metric as follows (Adamiak, 2005).

$$ds^2 = -dt^2 + \frac{dr^2}{1-b(r)/r} + r^2[d\theta^2 + \sin^2\theta\, d\phi^2] \tag{23}$$

Using Eqs. (10) and (22), the energy distributions of zero radial tides wormhole in general relativity and teleparallel gravity are obtained

$$E^{GR} = E^{TG} = 0. \tag{24}$$

3.2. Møller Energy of Conformal Wormhole in GR and TG

Putting $A^2(r,t) = \Omega^2(t)$, $B^2(r,t) = \frac{\Omega^2(t)}{1-b(r)/r}$ and $C^2(r,t) = \Omega^2(t)$ in the line element given by Eq. (3), we get the conformal wormhole metric as follows (Kar, 1994; Kar and Sadhev, 1996).

$$ds^2 = \Omega^2(t)\left[-dt^2 + \frac{dr^2}{1-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{25}$$

where Ω is the conforrmal factor, a smooth, finite and strictly positive function (Wald, 1984). Conformal wormholes could be considered one of the popular approaches to evolving wormholes. Conformal transformation of the wormhole was considered by Kar (1994) in order to find out if within classical general relativity a class of nonstable not violating energy conditions wormholes could exist. It was found that evolving geometry can support a wormhole.

The conformal transformation technique is used in general for bringing the points at infinity to a finite position and hence analyze the causal structure of infinity.

From Eqs. (10), (22) and (25), we find the energy of conformal wormholes in general relativity and teleparallel gravity

$$E^{GR} = E^{TG} = 0. \tag{26}$$

3.3. Møller Energy of Zero Density Wormhole in GR and TG

Replacing $A^2(r,t) = U(r)$, $B^2(r,t) = \frac{1}{1-r_0/r}$ and $C^2(r,t) = 1$ in the line element given by Eq. (3), nonstatic spherically symmetric metric becomes to the "zero density" wormhole metric. In this model, we set the shape function to be constant $b(r) = r_0$. By substituting $r_0 = 2M$ one can make the observation that the metric is analogous to non-traversable Schwarzschild blackhole, but since we do not allow $U(r)$ to slip to infinity, this configuration is traversable. Thus the metric of "zero density" wormhole is described by

$$ds^2 = -U(r)\,dt^2 + \frac{dr^2}{1 - r_0/r} + r^2[d\theta^2 + \sin^2\theta d\phi^2] \tag{27}$$

For simple case of constant $U(r)$ the wormhole needs twice less exotic matter at the throat than in "zero tidal" example (Adamiak, 2005).

Using Eqs. (10), (22) and (27), we have the following energy distribution for the "zero density" wormhole

$$E^{GR} = E^{TG} = \frac{U_{,r}\, r^2\sqrt{1 - \frac{r_0}{r}}}{2\sqrt{U}}. \tag{28}$$

3.4. Møller Energy of Inflating Wormhole in GR and TG

If we put $A^2(r,t) = U(r)$, $B^2(r,t) = \frac{e^{2\chi t}}{1-b(r)/r}$ and $C^2(r,t) = e^{2\chi t}$ in the line element given by Eq. (3), non-static spherically symmetric metric transforms to the inflating wormhole metric.

$$ds^2 = -U(r)\,dt^2 + e^{2\chi t}\left[\frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{29}$$

where $\chi = \sqrt{\Lambda/3}$ and Λ is the cosmological constant (Qadir, 1986). Roman (1993) was to explore the possibility that inflation might provide a natural mechanism for the enlargement of such wormholes to macroscopic size. A new classical metric was presented for a Lorentzian wormhole which is imbedded in a flat deSitter space. It was shown that the throat and the proper length of the wormhole inflate. The metric can be obtained by multiplication of the special part of Morris–Thorne wormhole by deSitter scale factor $e^{2\chi t}$.

From Eqs. (10) and (22), the energy of the inflating wormhole is obtained as

$$E^{GR} = E^{TG} = \frac{U_{,r} e^{\chi t} r^2\sqrt{1 - \frac{b(r)}{r}}}{2\sqrt{U}}. \tag{30}$$

These solutions correspond to the flat cosmological model wormholes (See Eq. (32) for $k = 0$). If $t \to 0$ and $b(r) = r_0$, our solutions reduce to zero density wormhole's energy given by Eq. (28).

3.5. Møller Energy of Wormhole Models in FRW Cosmology in GR and TG

If we replace $A^2(r,t) = U(r)$, $B^2(r,t) = \frac{R^2(t)}{1-kr-b(r)/r}$ and $C^2(r,t) = R^2(t)$ in the line element given by Eq. (3), non-static spherically symmetric metric transforms to the wormhole metric in FRW cosmology. FRW space-time with static wormhole metric (Kim, 1996) is given by

$$ds^2 = -U(r)\,dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2-b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \quad (31)$$

where $R(t)$ is the scale factor of the universe and k is $-1, 1, 0$, the sign of the curvature of space-time.

The wormhole space-time represents two open universes connected by Lorentzian wormhole and has the following features (Li-Xin Li, 2001): (i) It can be exactly solve the Einstein Equations, (ii) The weak energy condition is satisfied everywhere, (iii) It has a topology of $R^2 \times T_g$ $(g \geq 2)$, (iv) It has no event horizons. The wormhole space-time is constructed from a usual open FRW universe. An open FRW universe has negative spatial curvature and can be foliated with spatial hyperbolic hyper-surfaces. So, we can write the useful metric constructed a cosmological wormhole by combining two space-time metrics:

Using Eqs. (10), (22) and (31), the energies of the cosmological wormholes or wormhole models in FRW cosmology are given by

$$E^{GR} = E^{TG} = \frac{U_{,r}r^2\sqrt{1-kr^2-\frac{b(r)}{r}}}{2\sqrt{U}}. \quad (32)$$

When $t \to 0$ in Eq. (30) and $k = 0$ in Eq. (32), our solutions reduce to inflating wormhole's energy.

3.6. Møller Energy of Visser–Kar–Dadich Wormhole in GR and TG

If we replace $A^2(r,t) = U(r)$, $B^2(r,t) = \frac{1}{1-2m/r}$ and $C^2(r,t) = 1$ in the line element given by Eq. (3), non-static spherically symmetric metric transforms to the Visser–Kar and Dadich (VKD) wormhole metric (Visser et al., 2003).

$$ds^2 = -U(r)\,dt^2 + \left[\frac{dr^2}{1-2m/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \quad (33)$$

VKD then considered a class of wormholes for which the "averaged null energy condition ANEC" line integral is finite and negative, but for which the volume integral above can be made as small as one likes. They concluded that one can construct traversable wormholes using only arbitrarily small amounts of exotic matter.

Table I. The Møller Energy of Various Wormholes in GR and TG

Wormholes	Møller Energy in GR	Møller Energy in TG
ZeroRadialTidesWormhole	$E^{GR}=0$	$E^{TG}=0$
ConformalWormhole	$E^{GR}=0$	$E^{TG}=0$
Zero Density Wormhole	$E^{GR}=\frac{U_{,r}r^2\sqrt{1-\frac{r_0}{r}}}{2\sqrt{U}}$	$E^{TG}=\frac{U_{,r}r^2\sqrt{1-\frac{r_0}{r}}}{2\sqrt{U}}$
Inflating Wormhole	$E^{GR}=\frac{U_{,r}e^{\chi t}r^2\sqrt{1-\frac{b(r)}{r}}}{2\sqrt{U}}$	$E^{TG}=\frac{U_{,r}e^{\chi t}r^2\sqrt{1-\frac{b(r)}{r}}}{2\sqrt{U}}$
Cosmological Wormhole	$E^{GR}=\frac{U_{,r}r^2\sqrt{1-kr^2-\frac{b(r)}{r}}}{2\sqrt{U}}$	$E^{TG}=\frac{U_{,r}r^2\sqrt{1-kr^2-\frac{b(r)}{r}}}{2\sqrt{U}}$
VKD Wormhole	$E^{GR}=\frac{U_{,r}r^2\sqrt{1-\frac{2m}{r}}}{2\sqrt{U}}$	$E^{TG}=\frac{U_{,r}r^2\sqrt{1-\frac{2m}{r}}}{2\sqrt{U}}$

All of the models discussed by VKD are "spatially Schwarzschild," that is,

$$b(r) = 2m = r_0 \tag{34}$$

so the $t = const$ spatial slices are the same as those Schwarzschild.

In this case, energies of VKD wormhole are given by

$$E^{GR} = E^{TG} = \frac{U_{,r}r^2\sqrt{1-\frac{2m}{r}}}{2\sqrt{U}}. \tag{35}$$

If $b(r) = 2m = r_0$, these solutions correspond to zero density wormhole's energy.

4. SUMMARY AND DISCUSSION

The tetrad definition of a gravitational field equation was maintained a more satisfactory treatment of the energy-momentum complex than does general relativity by Møller. Accordingly, we have also applied the super-potential method of Mikhail *et al.* (1993) to calculate the energy of the well-known wormholes.

In this paper, we considered Møller energy-momentum definition in both general relativity and teleparallel gravity in order to investigate the energy associated with various wormhole space-times such as zero radial tides wormhole zero density wormhole, conformal wormhole, inflating wormhole, cosmological wormhole and Visser–Kar–Dadich wormhole space-times using the non-static spherically symmetric metric.

Although Møller energy definition is different in GR and TG, we obtained that energy distribution is the same in both of these different gravitation theories.

In the case of Møller energies of zero redial tides wormhole and conformal wormhole, we obtain that these energy definitions are identical not only general but also teleparallel gravity. Also, we get that total energy densities are vanishing everywhere in GR and TG. This means that the energy contributions from matter and gravitational field inside an arbitrary two-surface cancel each other.

Moreover, it is independent of the teleparallel dimensionless coupling constant, which means that it is valid not only in teleparallel equivalent of general relativity but also in any teleparallel model.

In the case of Møller energies of zero density wormhole, inflating wormhole, cosmological wormhole and VKD wormhole, we find the same energy distributions which are different from zero for these space-time in GR and TG. From these, we have concluded that the energy distributions are dependent of the teleparallel dimensionless coupling constants, which means that it is valid only in the teleparallel equivalent of general relativity, it is not valid teleparallel model. Hence, one can perform the calculations and get the same energy distributions in the general relativity.

Our results given by Eqs. (9) and (10) agree with Xulu's (2003) results. Also, our results given by Eqs. (9) and (10) contain Salti's (2006) results, i.e., the static wormhole, wormhole with electric charge and scalar field.

Finally, our results support Lessner's (1996) view that Møller energy-momentum complex is the powerful concept to calculate energy distribution in a given space-times.

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